Thermodynamics of interacting holographic dark energy with apparent horizon as an IR cutoff

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As soon as an interaction between holographic dark energy and dark matter is taken into account, the identification of IR cutoff with Hubble radius H^{-1}, in flat universe, can simultaneously drive accelerated expansion and solve the coincidence problem. Based on this, we demonstrate that in a non-flat universe the natural choice for IR cutoff could be the apparent horizon radius, $\tilde{r}_A = 1/\sqrt{H^2 + k/a^2}$. We show that any interaction of dark matter with holographic dark energy, whose infrared cutoff is set by the apparent horizon radius, implies an accelerated expansion and a constant ratio of the energy densities of both components thus solving the coincidence problem. We also verify that for a universe filled with dark energy and dark matter the Friedmann equation can be written in the form of the modified first law of thermodynamics, $dE = T_h dS_h + W dV$, at apparent horizon. In addition, the generalized second law of thermodynamics is fulfilled in a region enclosed by the apparent horizon. These results hold regardless of the specific form of dark energy and interaction term. Our study might reveal that in an accelerating universe with spatial curvature, the apparent horizon is a physical boundary from the thermodynamical point of view.

I. INTRODUCTION

The combined analysis of cosmological observations reveal that nearly three quarters of our universe consists of a mysterious energy component usually dubbed "dark energy" which is responsible for the cosmic expansion, and the remaining part consists of pressureless matter [1]. The nature of such previously unforeseen energy still remains a complete mystery, except for the fact that it has negative pressure. In this new conceptual set up, one of the important questions concerns the thermodynamical behavior of the accelerated expanding universe driven by dark energy. It is important to ask whether thermodynamics in an accelerating universe can reveal some properties of dark energy. The profound connection between thermodynamics and gravity has been observed in the cosmological situations [2, 3, 4, 5, 6, 7, 8, 9, 10]. This connection implies that the thermodynamical properties can help understand the dark energy, which gives strong motivation to study

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thermodynamics in the accelerating universe. It is also of great interest to investigate the validity of the generalized second law of thermodynamics in the accelerating universe driven by dark energy [11]. The generalized second law of thermodynamics is an important principle in governing the development of the nature.

An interesting attempt for probing the nature of dark energy within the framework of quantum gravity, is the so-called "Holographic Dark Energy" (HDE) proposal. This model which has arisen a lot of enthusiasm recently [12, 13, 14, 15, 16, 17, 18, 19], is motivated from the holographic hypothesis [20] and has been tested and constrained by various astronomical observations [21]. It is important to note that in the literature, various scenarios of HDE have been studied via considering different system's IR cutoff. In the absence of interaction between dark matter and dark energy in flat universe, Li [13] discussed three choices for the length scale L which is supposed to provide an IR cutoff. The first choice is the Hubble radius, $L = H^{-1}$ [15], which leads to a wrong equation of state, namely that for dust. The second option is the particle horizon radius. In this case it is impossible to obtain an accelerated expansion. Only the third choice, the identification of L with the radius of the future event horizon gives the desired result, namely a sufficiently negative equation of state to obtain an accelerated universe.

However, as soon as an interaction between dark energy and dark matter is taken into account, the first choice, $L = H^{-1}$, in flat universe, can simultaneously drive accelerated expansion and solve the coincidence problem [22]. Based on this, we demonstrate that in a non-flat universe the natural choice for IR cutoff could be the apparent horizon radius. We show that any interaction of pressureless dark matter with HDE, whose infrared cutoff is set by the apparent horizon radius, implies a constant ratio of the energy densities of both components thus solving the coincidence problem. Besides, it was argued that for an accelerating universe inside the event horizon the generalized second law does not satisfy, while the accelerating universe enveloped by the Hubble horizon satisfies the generalized second law [23]. This implies that the event horizon in an accelerating universe might not be a physical boundary from the thermodynamical point of view. Thus, it looks that we need to define a convenient horizon that satisfies all of our accepted principles in a universe with any spacial curvature. In the next section, we study the interacting HDE with apparent horizon as an IR cutoff. In section III, we examine the first law of thermodynamics on the apparent horizon in an accelerating universe with spacial curvature. In section IV, we investigate the validity of the generalized second law of thermodynamics in a region enclosed by the apparent horizon. The last section is devoted to conclusions.

II. INTERACTING HDE WITH APPARENT HORIZON AS AN IR CUTOFF

We consider a homogenous and isotropic Friedmann-Robertson-Walker (FRW) universe which is described by the line element

$$ds^2 = h_{\mu\nu}dx^\mu dx^\nu + \tilde{r}^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $\tilde{r} = a(t)r$, $x^0 = t, x^1 = r$, the two dimensional metric $h_{\mu\nu}$=diag $(-1, a^2/(1-kr^2))$. Here k denotes the curvature of space with $k = 0, 1, -1$ corresponding to open, flat, and closed universes, respectively. A closed universe with a small positive curvature ($\Omega_k \simeq 0.01$) is compatible with observations [24]. Then, the dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is determined by the relation $h^{\mu\nu}\partial_\mu\tilde{r}\partial_\nu\tilde{r} = 0$, which implies that the vector $\nabla\tilde{r}$ is null on the apparent horizon surface. The apparent horizon was argued as a causal horizon for a dynamical spacetime and is associated with gravitational entropy and surface gravity [25, 26]. A simple calculation gives the apparent horizon radius for the FRW universe

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}. \tag{2}$$

The corresponding Friedmann equation takes the form

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_m + \rho_D\right), \tag{3}$$

where ρ_m and ρ_D are the energy density of dark matter and dark energy inside apparent horizon, respectively. Since we consider the interaction between dark matter and dark energy, ρ_m and ρ_D do not conserve separately; they must rather enter the energy balances

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{4}$$

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q. \tag{5}$$

where $w_D = p_D/\rho_D$ is the equation of state parameter of HDE, and Q stands for the interaction term. We also ignore the baryonic matter ($\Omega_{BM} \approx 0.04$) in comparison with dark matter and dark energy ($\Omega_{DM} + \Omega_{DE} \approx 0.96$). We shall assume the ansatz $Q = \Gamma\rho_D$ with $\Gamma > 0$ which means that there is an energy transfer from the dark energy to dark matter. It is important to note that the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor H) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i) $Q \propto H\rho_D$, (ii) $Q \propto H\rho_m$, or (iii) $Q \propto H(\rho_m + \rho_D)$. However, we can present the above three forms

in one expression as $Q = \Gamma\rho_D$, where

$$\begin{aligned}
&\Gamma = 3b^2 H &&\text{for}\ \ Q \propto H\rho_D, \\
&\Gamma = 3b^2 Hu &&\text{for}\ \ Q \propto H\rho_m, \\
&\Gamma = 3b^2 H(1+u) &&\text{for}\ \ Q \propto H(\rho_m + \rho_D),
\end{aligned} \tag{6}$$

with b^2 is a coupling constant and $u = \rho_m/\rho_D$ is the ratio of energy densities. The freedom of choosing the specific form of the interaction term Q stems from our incognizance of the origin and nature of dark energy as well as dark matter. Moreover, a microphysical model describing the interaction between the dark components of the universe is not available nowadays. If we introduce, as usual, the fractional energy densities such as

$$\Omega_m = \frac{8\pi G\rho_m}{3H^2}, \qquad \Omega_D = \frac{8\pi G\rho_D}{3H^2}, \qquad \Omega_k = \frac{k}{H^2a^2}, \tag{7}$$

then, the Friedmann equation can be written as $\Omega_m + \Omega_D = 1 + \Omega_k$. In terms of the apparent horizon radius, we can rewrite the Friedmann equation as

$$\frac{1}{\tilde{r}_A^2} = \frac{8\pi G}{3}\left(\rho_m + \rho_D\right). \tag{8}$$

For completeness, we give the deceleration parameter

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}, \tag{9}$$

which combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. It is a matter of calculation to show that

$$q = -(1+\Omega_k) + \frac{3}{2}\Omega_D(1+u+w_D). \tag{10}$$

The evolution of u is governed by

$$\dot{u} = 3Hu\left[w_D + \frac{1+u}{u}\frac{\Gamma}{3H}\right]. \tag{11}$$

We assume the HDE density has the form

$$\rho_D = \frac{3c^2}{8\pi G\tilde{r}_A^2}, \tag{12}$$

where c^2 is a constant, the coefficient 3 is for convenient, and we have set the apparent horizon radius $L = \tilde{r}_A$ as system's IR cutoff in holographic model of dark energy. Inserting Eq. (12) in Eq. (8) immediately yields

$$\rho_m = \frac{3(1-c^2)}{8\pi G\tilde{r}_A^2}. \tag{13}$$

Thus we reach

$$u = \frac{\rho_m}{\rho_D} = \frac{1-c^2}{c^2}. \tag{14}$$

This implies that the ratio of the energy densities is a constant; thus the coincidence problem can be solved. Taking the derivative of Eq. (12) we get

$$\dot{\rho}_D = -2\rho_D \frac{\dot{\tilde{r}}_A}{\tilde{r}_A} = -3c^2 H \rho_D (1 + u + w_D). \tag{15}$$

where we have employed Eqs. (4), (5) and (8). Combining this equation with (5) we obtain

$$w_D = -\left(1 + \frac{1}{u}\right)\frac{\Gamma}{3H}. \tag{16}$$

Substituting w_D into (10), we find

$$q = -(1+\Omega_k) - \frac{3}{2}\Omega_D(1+u)\left(\frac{\Gamma}{3Hu} - 1\right). \tag{17}$$

The interaction parameter $\frac{\Gamma}{3H}$ together with the energy density ratio u determine the equation of state parameter. In the absence of interaction, we encounter dust with $w_D = 0$. For the choice $L = \tilde{r}_A$ an interaction is the only way to have an equation of state different from that for dust. Any decay of the dark energy component into pressureless matter is necessarily accompanied by an equation of state $w_D < 0$. The existence of an interaction has another interesting consequence. Inserting expression w_D into (11) leads to $\dot{u} = 0$, i.e., $u = \text{const}$. Thus, any interaction of dark matter with HDE, whose infrared cutoff is set by the apparent horizon radius, implies an accelerated expansion and a constant ratio of the energy densities, irrespective of the specific structure of the interaction. It is important to note that although choosing $L = H^{-1}$, in a spatially flat universe, can drive accelerated expansion and solve the coincidence problem [22], but taking into account the spatial curvature term gives rise to an additional dynamics which implies a small (compared with the Hubble rate) change of the energy density ratio; thus the coincidence problem cannot be solved exactly (see [28] for details). This implies that in an accelerating universe with spacial curvature the Hubble radius H^{-1} is not a convenient choice.

In summary, in a universe with spacial curvature, the identification of IR cutoff with apparent horizon radius $\tilde{r}_A$ is not only the most obvious but also the simplest choice which can simultaneously drive accelerated expansion and solve the coincidence problem. It is important to note that the interaction is essential to simultaneously solve the coincidence problem and have late acceleration. There is no non-interacting limit, since in the absence of interaction, i.e., $\Gamma = 0$, there is no acceleration.

III. FIRST LAW OF THERMODYNAMICS

In this section we are going to examine the first law of thermodynamics. In particular, we show that for a closed universe filled with HDE and dark matter the Friedmann equation can be written directly in the form of the modified first law of thermodynamics at apparent horizon regardless of the specific form of the dark energy. The associated temperature with the apparent horizon can be defined as $T = \kappa/2\pi$, where κ is the surface gravity $\kappa = \frac{1}{\sqrt{-h}}\partial_\mu\left(\sqrt{-h}h^{\mu\nu}\partial_{\mu\nu}\tilde{r}\right)$. Then one can easily show that the surface gravity at the apparent horizon of FRW universe can be written as

$$\kappa = -\frac{1}{\tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{18}$$

When $\dot{\tilde{r}}_A \leq 2H\tilde{r}_A$, the surface gravity $\kappa \leq 0$, which leads the temperature $T \leq 0$ if one defines the temperature of the apparent horizon as $T = \kappa/2\pi$. Physically it is not easy to accept the negative temperature, the temperature on the apparent horizon should be defined as $T = |\kappa|/2\pi$. Recently the connection between temperature on the apparent horizon and the Hawking radiation has been considered in [29], which gives more solid physical implication of the temperature associated with the apparent horizon.

Taking differential form of equation (8) and using Eqs. (4) and (5), we can get the differential form of the Friedmann equation

$$\frac{1}{4\pi G}\frac{d\tilde{r}_A}{\tilde{r}_A^3} = H\rho_D\left(1 + u + w_D\right)dt. \tag{19}$$

Multiplying both sides of the equation (19) by a factor $4\pi\tilde{r}_A^3\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)$, and using the expression (18) for the surface gravity, after some simplification one can rewrite this equation in the form

$$-\frac{\kappa}{2\pi}\frac{2\pi\tilde{r}_A d\tilde{r}_A}{G} = 4\pi\tilde{r}_A^3 H\rho_D\left(1 + u + w_D\right)dt - 2\pi\tilde{r}_A^2\rho_D\left(1 + u + w_D\right)d\tilde{r}_A. \tag{20}$$

$E = (\rho_m + \rho_D)V$ is the total energy content of the universe inside a 3-sphere of radius $\tilde{r}_A$, where $V = \frac{4\pi}{3}\tilde{r}_A^3$ is the volume enveloped by 3-dimensional sphere with the area of apparent horizon $A = 4\pi\tilde{r}_A^2$. Taking differential form of the relation $E = (\rho_m + \rho_D)\frac{4\pi}{3}\tilde{r}_A^3$ for the total matter and energy inside the apparent horizon, we get

$$dE = 4\pi\tilde{r}_A^2(\rho_m + \rho_D)d\tilde{r}_A + \frac{4\pi}{3}\tilde{r}_A^3(\dot{\rho}_m + \dot{\rho}_D)dt. \tag{21}$$

Using Eqs. (4) and (5), we obtain

$$dE = 4\pi\tilde{r}_A^2\rho_D(1 + u)d\tilde{r}_A - 4\pi\tilde{r}_A^3 H\rho_D\left(1 + u + w_D\right)dt. \tag{22}$$

Substituting this relation into (20), and using the relation between temperature and the surface gravity, we get the modified first law of thermodynamics on the apparent horizon

$$dE = T_h dS_h + W dV, \tag{23}$$

where $S_h = A/4G$ is the entropy associated to the apparent horizon, and

$$W = \frac{1}{2}(\rho_m + \rho_D - p_D) = \frac{1}{2}\rho_D\,(1 + u - w_D) \tag{24}$$

is the matter work density [25]. The work density term is regarded as the work done by the change of the apparent horizon, which is used to replace the negative pressure if compared with the standard first law of thermodynamics, $dE = TdS - pdV$. For a pure de Sitter space, $\rho_m + \rho_D = -p_D$, then our work term reduces to the standard $-p_D dV$ and we obtain exactly the first law of thermodynamics.

IV. GENERALIZED SECOND LAW OF THERMODYNAMICS

In this section we turn to investigate the validity of the generalized second law of thermodynamics in a region enclosed by the apparent horizon. Differentiating Eq. (8) with respect to the cosmic time and using Eqs. (4) and (5) we get

$$\dot{\tilde{r}}_A = 4\pi G H \tilde{r}_A^3 \rho_D (1 + u + w_D). \tag{25}$$

One can see from the above equation that $\dot{\tilde{r}}_A > 0$ provided condition $w_D > -1 - u$, holds. Let us now turn to find out $T_h \dot{S}_h$:

$$T_h \dot{S}_h = \frac{1}{2\pi \tilde{r}_A}\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)\frac{d}{dt}\left(\frac{\pi \tilde{r}_A^2}{G}\right). \tag{26}$$

After some simplification and using Eq. (25) we get

$$T_h \dot{S}_h = 4\pi H \tilde{r}_A^3 \rho_D (1 + u + w_D)\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right). \tag{27}$$

As we argued above the term $\left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A}\right)$ is positive to ensure $T_h > 0$, however, in an accelerating universe the equation of state parameter of dark energy may cross the phantom divide, i.e., $w_D < -1 - u$. This indicates that the second law of thermodynamics, $\dot{S}_h \geq 0$, does not hold on the apparent horizon. Then the question arises, "will the generalized second law of thermodynamics, $\dot{S}_h + \dot{S}_m + \dot{S}_D \geq 0$, can be satisfied in a region enclosed by the apparent horizon?" The entropy of dark energy plus dark matter inside the apparent horizon, $S = S_m + S_D$, can be related to the total energy $E = (\rho_m + \rho_D)V$ and pressure p_D in the horizon by the Gibbs equation [30]

$$TdS = d[(\rho_m + \rho_D)V] + p_D dV = V(d\rho_m + d\rho_D) + \rho_D(1 + u + w_D)dV, \tag{28}$$

where $T = T_m = T_D$ and $S = S_m + S_D$ are the temperature and the total entropy of the energy and matter content inside the horizon, respectively. Here we assumed that the temperature of both dark components are equal, due to their mutual interaction. We also limit ourselves to the assumption that the thermal system bounded by the apparent horizon remains in equilibrium so that the temperature of the system must be uniform and the same as the temperature of its boundary. This requires that the temperature T of the energy content inside the apparent horizon should be in equilibrium with the temperature T_h associated with the apparent horizon, so we have $T = T_h$[30]. This expression holds in the local equilibrium hypothesis. If the temperature of the fluid differs much from that of the horizon, there will be spontaneous heat flow between the horizon and the fluid and the local equilibrium hypothesis will no longer hold. This is also at variance with the FRW geometry. In general, when we consider the thermal equilibrium state of the universe, the temperature of the universe is associated with the apparent horizon. Therefore from the Gibbs equation (28) we can obtain

$$T_h(\dot{S_m} + \dot{S_D}) = 4\pi\tilde{r}_A^2\rho_D(1+u+w_D)\dot{\tilde{r}}_A - 4\pi H\tilde{r}_A^3\rho_D(1+u+w_D). \tag{29}$$

To check the generalized second law of thermodynamics, we have to examine the evolution of the total entropy $S_h + S_m + S_D$. Adding equations (27) and (29), we get

$$T_h(\dot{S}_h + \dot{S}_m + \dot{S}_D) = 2\pi\tilde{r}_A^2\rho_D(1+u+w_D)\dot{\tilde{r}}_A = \frac{A}{2}\rho_D(1+u+w_D)\dot{\tilde{r}}_A. \tag{30}$$

where $A > 0$ is the area of apparent horizon. Substituting $\dot{\tilde{r}}_A$ from Eq. (25) into (30) we get

$$T_h(\dot{S}_h + \dot{S}_m + \dot{S}_D) = 2\pi GAH\tilde{r}_A^3\rho_D^2(1+u+w_D)^2. \tag{31}$$

The right hand side of the above equation cannot be negative throughout the history of the universe, which means that $\dot{S_h} + \dot{S_m} + \dot{S_D} \geq 0$ always holds. This indicates that for a universe with spacial curvature filled with interacting dark components, the generalized second law of thermodynamics is fulfilled in a region enclosed by the apparent horizon.

V. CONCULUSIONS

It is worthwhile to note that in the literature, various scenarios of HDE have been studied via considering different system's IR cutoff. In the absence of interaction the convenient choice for the IR cutoff are the radial size of the horizon R_h and the radius of the event horizon measured on the sphere of the horizon $L = ar(t)$ in spatially flat and curved universe, respectively. Although,

in these cases the HDE gives the observation value of dark energy in the universe and can drive the universe to an accelerated expansion phase, but an obvious drawback concerning causality appears. Event horizon is a global concept of spacetime; existence of event horizon of the universe depends on future evolution of the universe; and event horizon exists only for universe with forever accelerated expansion. However, as soon as an interaction between dark energy and dark matter is taken into account, the identification of L with H^{-1} in flat universe, can simultaneously drive accelerated expansion and solve the coincidence problem [22]. The Hubble radius is not only the most obvious but also the simplest choice in flat universe.

In this paper, we demonstrated that in a universe with spacial curvature the natural choice for IR cutoff could be the apparent horizon radius, $\tilde{r}_A = 1/\sqrt{H^2+k/a^2}$. We showed that any interaction of pressureless dark matter with HDE, whose infrared cutoff is set by the apparent horizon radius, implies a constant ratio of the energy densities of both dark components thus solving the coincidence problem. In addition, we examined the validity of the first and the generalized second law of thermodynamics for a universe filled with mutual interacting dark components in a region enclosed by the apparent horizon. These results hold regardless of the specific form of dark energy and interaction term Q. Our study further supports that in a universe with spatial curvature, the apparent horizon is a physical boundary from the thermodynamical point of view.

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